FUNCTIONS OF SEVERAL VARIABLES AND THE DEFINITION OF THE PARTIAL DERIVATIVE

FUNCTIONS OF SEVERAL VARIABLES

$z = f(x,y)$

FUNCTIONS OF TWO VARIABLES

EXAMPLE:

$z = f(x,y) = x^2 + xy^2 + 2$

At $(x_0, y_0) = (-1, 3)$,

$z = f(-1,3) =$

$= (-1)^2 + (-1)(9) + 2$

$= 1 - 9 + 2 = -6$

$\boxed{z = f(-1,3) = -6}$

Also: $z\Big|_{(-1,3)} = -6$

The Domain of f

$= \{$ Set of Inputs! $\}$

$= \{ (x,y) \mid x, y \in \mathbb{R} \}$

$= \mathbb{R} \times \mathbb{R}$ = The x-y plane

$u = g(x,y,z)$

FUNCTIONS OF THREE VARIABLES

EXAMPLE:

$u = g(x,y,z) = xz + y e^{z}$

$u = g(2,3,4) = 8 + 3e^4$

$g(0,5,-1) = \frac{5}{e} = 0 + 5e^{-1}$

The Domain of g =

$\{ (x,y,z) \mid x, y, z \in \mathbb{R} \}$

$= \mathbb{R} \times \mathbb{R} \times \mathbb{R}$

= "THREE-SPACE"

THERE ARE MULTI-VARIABLE FUNCTIONS OF MORE THAN THREE VARIABLES ALSO.

THE DEFINITION OF THE PARTIAL DERIVATIVES OF A FUNCTION OF <u>TWO</u> VARIABLES:

[THE DEFINITIONS FOR FUNCTIONS OF THREE OR MORE VARIABLES ARE SIMILAR.]

Given the function $z = f(x, y)$, its PARTIAL DERIVATIVES f_x and f_y, as defined at the point (x_0, y_0), are as follows:

$$f_x(x_0, y_0) = \lim_{h \to 0} \frac{f(x_0+h, y_0) - f(x_0, y_0)}{h}$$

[where y is held fixed at $y = y_0$]

AND

$$f_y(x_0, y_0) = \lim_{h \to 0} \frac{f(x_0, y_0+h) - f(x_0, y_0)}{h}$$

[where x is held fixed at $x = x_0$].

The Partial Derivative function f_x of f with respect to x is

$$f_x(x, y) = \lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h}$$ which is

the derivative of f where all the variables other than x are held constant.

The Partial Derivative function f_y of f with respect to y is

$$f_y(x, y) = \lim_{h \to 0} \frac{f(x, y+h) - f(x, y)}{h}$$ which is

the derivative of f where all the variables other than y are held constant.